



NON PERTURBATIVE EFFECTS IN QCD

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Non perturbative results from lattice QCD will be discussed, namely : Vacuum Condensates and QCD Sum Rules; $U_A(1)$ and Topology; Confinement of Color.

1 INTRODUCTION

QCD is usually quantized perturbatively.

The Lagrangean L is split into the sum of two terms

$$L = L_0 + L_I \quad (1)$$

where L_0 describes free quarks and gluons, and L_I their interaction. Then L_0 is quantized : the Hilbert space is the Fock space of free quarks and gluons and the interaction describes scattering between them.

Any observable $\langle O \rangle$ is computed as a power series expansion in the renormalized coupling constant α_s

$$\langle O \rangle = \sum_n O_n \alpha_s^n \quad (2)$$

where O_n are finite amplitudes.

The expansion eq(2) is not convergent, not even as an asymptotic series[1]. Nevertheless it is an empirical fact that, as $q^2 \rightarrow \infty$ or for $q^2 > 1\text{GeV}^2$ $\alpha_s \ll 1$ and a few terms of the expansion describe physics correctly. The lack of convergence of the expansion eq(2) reflects the instability of Fock vacuum. Quarks and gluons are confined and therefore free quarks and gluons are not a good zeroth order approximation to physics.

A non perturbative quantization of the theory is needed to identify the true vacuum state. This is provided by the Feynman path integral formulation. The key quantity is the partition function Z , in terms of which all field correlators can be computed.

$$Z = \int [dA_\mu][d\psi][d\bar{\psi}] \exp - [S(A_\mu, \bar{\psi}, \psi)] \quad (3)$$

Z in eq(4) is a functional integral and is defined by approximating euclidean space time by a discrete set of points in a finite volume, so that the number of integration variables is finite and the integral is an ordinary well defined integral. A sequence of such integrals is then computed on sets of points which tend to cover the volume densely. The limit of infinite volume

is then performed. If Z is finite and well defined after these limiting procedures one says that the theory exists as a field theory. The procedure also ensures the existence of the analytic continuation to Minkowskian space time.

For QCD asymptotic freedom insures that the first of these limits exists. The existence of a mass gap in the theory also insures that the infinite volume limit exists. A fully rigorous mathematical proof of these statements still does not exist, but it is physically reasonable to say that QCD exists as a field theory.

Lattice formulation of QCD is nothing but an approximant of Feynman integral in the sequence which defines it : at sufficiently low value of the unrenormalized coupling and at sufficiently large volume it will provide the physical amplitudes with any required precision from first principles (modulo technical computational difficulties).

Non perturbative phenomenology is based on Wilson's Operator Product Expansion (OPE) ,

$$T(A(x)B(0)) = \sum_n C_n(x) O_n(0) \quad (4)$$

where C_n 's describe short distances and are usually computed in perturbation theory and the operators O_n describe large distances. The sum in eq(4) is ordered by increasing order of the dimensions in mass of the local operators O_n .

Vacuum expectation values (vev) of eq(4) involve in the rhs vev's of composite operators (condensates) . They enter in the SVZ sum rules[2]. The gluon condensate $\langle G_2 \rangle = \langle 0 | \beta(g_s) / g_s G_{\mu\nu}^a(0) G_{\mu\nu}^a(0) | 0 \rangle$, and the quark condensate $\langle G_{\bar{\psi}\psi} \rangle = \langle 0 | \bar{\psi}(0) \psi(0) | 0 \rangle$ are examples of them. A condensate like G_2 has dimension 4 in mass, and therefore, by renormalization group arguments is related to the running coupling constant at the scale μ as $G_2 = \mu^4 \exp(-4/b_0/g^2(\mu^2))$ which is non analytic in g and undefined in perturbation theory.

Matrix elements of eq(4) between quark states are relevant to weak interaction matrix elements at distances

$\cong 1/M_W, 1/M_Z$, or to structure functions of deep inelastic scattering.

In this talk I will concentrate on the vacuum state, namely on condensates (sect 2), topology (sect3), and confinement of color (sect4)

2 QCD SUM RULES. CONDENSATES

The OPE of the product of two conserved currents

$$T(j_\mu(x)j_\nu(0)) = (g_{\mu\nu} - \partial_\mu\partial_\nu)\tilde{\Pi}(x^2) \quad (5)$$

taken on the vacuum reads

$$\langle \tilde{\Pi}(x^2) \rangle \sim \tilde{C}_I^*(x^2)\langle I \rangle + \tilde{C}_G(x^2)\langle G_2 \rangle + \tilde{C}_{\bar{\psi}\psi}(x^2)\langle G_{\bar{\psi}\psi} \rangle + \dots \quad (6)$$

where I is the identity operator ($\langle I \rangle = 1$), and at short distances $C_I(x^2) \sim 1/x^4$ modulo logs, C_G and $C_{\bar{\psi}\psi} \sim \text{const}$ modulo logs. Eq(6) is a theorem in perturbation theory, an assumption in the presence of non perturbative effects. The coefficients \tilde{C} describe short distances, the condensates describe large distance physics.

After Fourier transform

$$\begin{aligned} \Pi(q^2) - \Pi(0) &\equiv \int d^4x \langle \tilde{\Pi}(x^2) \rangle (\exp(iqx) - 1) \sim \\ C_I(q^2) + C_{G_2}(q^2)\langle G_2 \rangle + C_{\bar{\psi}\psi}(q^2)\langle G_{\bar{\psi}\psi} \rangle \end{aligned} \quad (7)$$

with

$$C_I(q^2) \sim \text{const} \quad (\text{modulo logs}) \quad (8)$$

$$C_{G_2}(q^2), C_{\bar{\psi}\psi}(q^2) \sim 1/q^4 \quad (\text{modulo logs}) \quad (9)$$

A dispersive representation for the l.h.s. of eq(7), if j_μ is the electromagnetic current is

$$\Pi(q^2) - \Pi(0) = -q^2 \int d\mu^2 \frac{R(\mu^2)}{\mu^2(q^2 - \mu^2 + i\epsilon)} \quad (10)$$

where $R(s) \equiv \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}$ can be taken from experiment.

An appropriate average on q^2 of both members of eq(10) gives the SVZ[2] sum rules, in which non perturbative effects are parametrized in terms of the condensates $\langle G_2 \rangle$ and $\langle G_{\bar{\psi}\psi} \rangle$. The result is a good phenomenology and a determination of the condensates

$$\langle G_2 \rangle = (.024 \pm .011) \text{GeV}^4, \quad (11)$$

$$\langle G_{\bar{\psi}\psi} \rangle \sim -.13 \text{GeV}^3 (q^2 = 1 \text{GeV}^2) \quad (12)$$

However the perturbative expansion of $\Pi(q^2) - \Pi(0)$ which provides the coefficient $C_I(q^2)$, when resummed at higher orders, is ambiguous by terms $\propto 1/q^4$ [1],[3] which mimic the terms with G_2 and $G_{\bar{\psi}\psi}$. The definition of the condensates is then intrinsically ambiguous. They could be defined as the coefficients of the terms $1/q^4$ in the expansion eq(7), but a priori, due to the ambiguity, they could be process dependent, i.e. dependent on the currents under consideration.

An OPE of the levels of bound $\bar{Q}Q$ energy levels [4][5] can be also performed. This amounts to compute the quadratic Stark effect produced on the levels by the fluctuating vacuum chromoelectric field. If the finite correlation length of the field strength correlators is taken into account [6][7] the relevant quantity is the gauge invariant correlator [8]

$$\langle G_2(x) \rangle \equiv \langle 0 | \text{Tr} \left\{ \vec{E}(x) U_C(x, 0) \vec{E}(0) U^\dagger(x, 0) \right\} | 0 \rangle \quad (13)$$

with $U_C(x, 0)$ the parallel transport from 0 to x along the path C , or $U_C(x, 0) = P \exp(i \int_C A_\mu(x) dx^\mu)$. A general parametrization of such correlators is [9][10]

$$\begin{aligned} D_{\mu\nu\rho\sigma}(x) &= \langle 0 | \text{Tr} [G_{\mu\nu}(x) U_C(x, 0) G_{\rho\sigma}(0) U_C^\dagger(x, 0)] | 0 \rangle = \\ &= (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) [D(x^2) + D_1(x^2)] + \\ &+ [x_\mu x_\rho g_{\nu\sigma} - x_\mu x_\sigma g_{\nu\rho} + x_\nu x_\sigma g_{\mu\rho} - x_\nu x_\rho g_{\mu\sigma}] \frac{\partial D_1(x^2)}{\partial x^2} \end{aligned} \quad (14)$$

In this language $\langle G_2(x) \rangle = \alpha/\pi(D + D_1)$. The correlators $D_{\mu\nu\rho\sigma}$ can be computed on the lattice [8][11] and with them their OPE. Their typical behaviour is shown in figs 1 and 2. A convenient parametrization of the Lattice data D^L, D_1^L is [11][12]

$$\frac{1}{a^4} D^L \underset{x \rightarrow 0}{\simeq} \frac{C_I}{x^4} + C_G \langle G_2 \rangle \exp(-x/\lambda) \quad (15)$$

$$\frac{1}{a^4} D_1^L \underset{x \rightarrow 0}{\simeq} \frac{C'_I}{x^4} + C'_G \langle G_2 \rangle \exp(-x/\lambda') \quad (16)$$

$\langle G_2(x) \rangle$ is a split point regulator of $\langle G_2 \rangle$. A best square fit to the lattice data gives, for pure gauge SU(3) [12] (quenched, no quarks) $G_2 = (.15 \pm .03) \text{GeV}^4$, $\lambda = \lambda' = (.22 \pm .03) \text{fm}$.

In full QCD [15] the same quantities can be computed. An extrapolation to realistic quark masses gives $\langle G_2 \rangle = (.022 \pm .005) \text{GeV}^4$ and $\lambda = \lambda' = (.32 \pm .04) \text{fm}$. The phenomenological value for $\langle G_2 \rangle$ is $(.024 \pm .005 \text{GeV}^4)$ [13]

From quenched to full QCD the gluon condensate decreases by almost an order of magnitude, and the correlation length increases by 50%.

Lattice vacuum correlators are the input of the Stochastic approach to QCD [10][14].

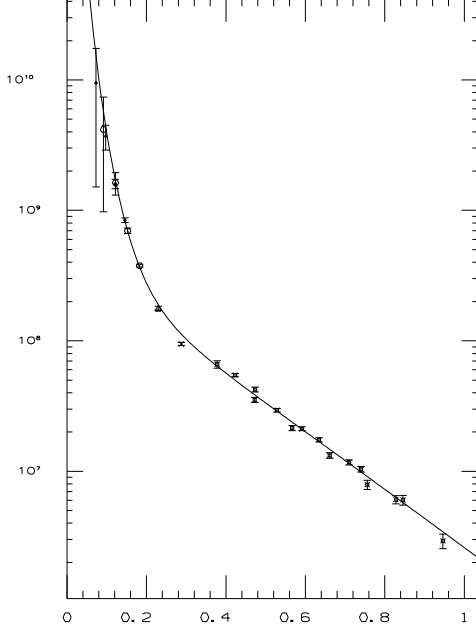


Figure 1. $D^L(x)$. The line is the best fit to eq(15).

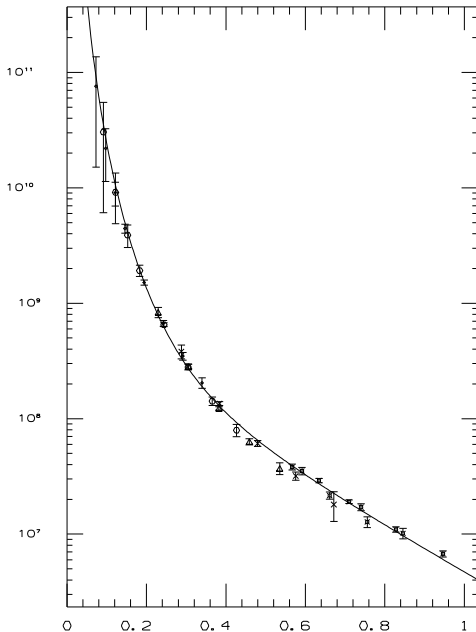


Figure 2. $D^L(x)_1$. The line is the best fit to eq(16).

In a similar way the fermion correlators can be computed[16]

$$\langle G_{\bar{\psi}\psi}(x) \rangle \equiv \langle 0 | \bar{\psi}(x) U_C(x, 0) \psi(0) | 0 \rangle \quad (17)$$

$\langle G_{\bar{\psi}\psi}(x) \rangle$ can be viewed as a split point regulator of $\langle G_{\bar{\psi}\psi} \rangle$.

Lattice data [16] give $\langle 0 | \bar{\psi}\psi | 0 \rangle$ consistent with the value obtained by use of SVZ sum rules and a correlation length $\lambda = (.42 \pm .05) fm$

3 TOPOLOGY

The $U_A(1)$ problem was a problem of the free quark model of Gellmann. The singlet axial current is conserved in the model, but in Nature the corresponding symmetry is neither realized a la Wigner (no parity doublets exist in the hadron spectrum), nor a la Goldstone : indeed a Goldstone-broken symmetry would imply [17] $m_{\eta'} < \sqrt{3}m_\pi$, which is badly violated by the observed value $m_{\eta'} \sim 980 Mev$. Hence the idea of Gellmann that the symmetry of hadrons could be abstracted from the free quark model was not correct for the axial singlet.

QCD solves the puzzle. The singlet axial current is anomalous in QCD and $U_A(1)$ is not a symmetry, being broken at the quantum level.

$$\partial_\mu j^\mu(x) = 2N_f Q(x) \quad (18)$$

$$Q(x) = \frac{1}{32\pi^2} G_{\mu\nu} G_{\mu\nu}^*$$

($G_{\mu\nu}^* \equiv 1/2 \epsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}$) is named topological charge density.

On continuous configurations the topological charge $Q = \int d^4x Q(x)$ is an integer (second Chern Number).

For quantized fields $Q(x)$ is an operator. The Topological susceptibility χ is defined as the response of the vacuum to $Q(x)$

$$\chi = \int d^4x \langle 0 | T(Q(x)Q(0)) | 0 \rangle \quad (19)$$

An argument based on $N_c \rightarrow \infty$ shows that $m_{\eta'}$ is related to the topological susceptibility of the $N_c = \infty$ vacuum.[18]

$$2N_f \chi = f_\pi^2 m_{\eta'}^2 (1 + O(1/N_c)) \quad (20)$$

As $N_c \rightarrow \infty$ dynamical quarks can be neglected and with the same approximation N_c can be taken equal to the physical value 3, so that χ is the topological

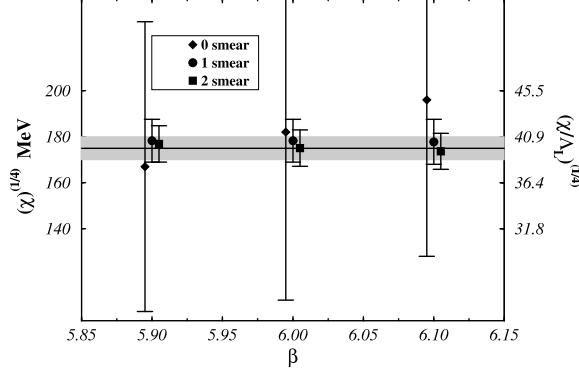


Figure 3. Determinations of χ by use of different lattice regulators and of different values of the unrenormalized coupling constant. The physical value of χ is independent of these choices within statistical errors.

susceptibility of the quenched SU(3) vacuum. An improved version of the argument including mixing to the octet gives[19]

$$2N_f\chi = f_\pi^2(m_{\eta'}^2 + m_\eta^2 - 2m_K^2)(1 + O(1/N_c)) \quad (21)$$

The prediction is in both cases $\chi \sim (180 \text{ MeV})^4$.

Computing χ from first principles can provide a cross check of QCD and of the $1/N_c$ expansion at the same time.

This problem has a long story: the main difficulty has been for a long time to understand how topology, which is based on continuity, can exist on the lattice which has a discrete structure. The way out is basic field theory[20] : any lattice version of the operator $Q(x)$, $Q_L(x)$, is related to the continuum counterpart, say in the \overline{MS} scheme, by a multiplicative renormalization

$$Q_L(x) = Z_Q Q(x) \quad (22)$$

The product $Q(x)Q(0)$ is singular as $x \rightarrow 0$. The OPE gives

$$Q(x)Q(0) \sim c_1(x)I + c_2(x)G_2 + \text{finite terms} \quad (23)$$

$$c_1 \sim x^{-8}, c_2 \sim x^{-4} \quad (24)$$

In computing χ an additive renormalization is also required.[21] In conclusion[22]

$$\chi = \frac{\chi_L - \chi_L^0}{Z^2} \quad (25)$$

where χ_L^0 is the lattice topological susceptibility of the $Q = 0$ sector of the vacuum. A technique developed

to compute Z_Q and χ_L^0 is known as "heating"[23]: it is based on the fact that topology has a much longer autocorrelation time τ_Q in monte-carlo upgrading than the autocorrelation time τ of the local quantum fluctuations which dominate the renormalization constants. Therefore χ_L^0 can be measured on the configurations obtained at times t such that $\tau \ll t \ll \tau_Q$ from a zero field configuration, which has topological charge zero. Similarly Q_L can be measured on configurations prepared from an initial configuration consisting of a single instanton, which has $Q=1$. At times long enough with respect to τ but very short compared to τ_Q Q_L will be equal to Z . This method is known as "field-theoretical". The result of this procedure is $\chi = ((175 \pm 5) \text{ MeV})^4$ [24] Recently an independent determination has been made, using the divergence of the singlet axial current operator, i.e. the lhs of eq (17). The advantage is in the use of a formulation of the fermions on the lattice which preserves chiral symmetry, and hence has multiplicative renormalization $Z_Q = 1$. The result is[25]

$$\chi = ((190 \pm 10) \text{ MeV})^4 \quad (26)$$

in complete agreement with the field theoretical determination. Also determinations based on cooling techniques [26] give consistent estimates. In conclusion the $U_A(1)$ problem is solved in QCD, and additional legitimation is added to the ideas of $N_c \rightarrow \infty$. An important issue, as we shall see in the next section, is the behaviour of χ at the deconfining phase transition.

4 CONFINEMENT OF COLOR

Quarks and gluons are visible at short distances, but have never been observed as free particles. This fact has led to the conjecture that colored particles never appear in asymptotic states, a property known as Confinement of Color. Confinement should be derivable from the QCD lagrangean.

Quarks have been searched in Nature since they were introduced by Gellman as fundamental constituents of hadrons[27], with negative results: only upper limits to their abundance and to their production rate have been established[28].

The ratio of quark abundance in nature, n_q to that of protons n_p has been given the upper limit

$$\frac{n_q}{n_p} \leq 10^{-27} \quad (27)$$

to be compared to the expectation based on the Standard Cosmological Model $\frac{n_q}{n_p} \approx 10^{-12}$.

As for production in particle reactions the best limit is provided by the inclusive cross section $\sigma_q \equiv \sigma(p + p \rightarrow q(\bar{q}) + X)$ namely

$$\sigma_q \leq 10^{-42} \text{cm}^2 \quad (28)$$

to be compared with the expectation, in the absence of confinement,

$$\sigma_q \simeq \sigma_{TOTAL} \approx 10^{-25} \text{cm}^2 \quad (29)$$

In both cases the ratio is depressed by a factor $\sim 10^{-15}$. The only natural explanation is that these ratios are exactly zero, i.e. that confinement is an absolute property, and as such based on some symmetry.

Experiments have been and are being performed to detect a deconfining transition at high energy density from hadronic matter to a state of free quarks and gluons (Quark Gluon Plasma), in high energy collisions of heavy ions[29]. The major difficulty with them is to identify an observable quantity by which deconfinement could be detected.

Some evidence for the existence of that transition has been produced by virtual experiments, namely by numerical simulations of QCD on a lattice.

The partition function of a field theory at temperature T is the euclidean Feynman path integral extending in euclidean time from $\tau = 0$ to $\tau = \frac{1}{T}$, with periodic boundary conditions for boson fields, antiperiodic for fermions.

For QCD this is computed on a lattice of spatial extension N_s^3 and temporal extension N_t with $N_t \ll N_s$, and the temperature is $T = \frac{1}{a(\beta)N_t}$ with $a(\beta)$ the lattice spacing in physical units, which can be tuned by varying the unrenormalized coupling constant $\beta \equiv \frac{2N_c}{g_s^2}$.

Also on the lattice, however, as in experiments, the real problem is to have a criterion to detect confinement.

In quenched formulation (pure gauge, no dynamical quarks) the criterion consists in looking at the large distance behaviour of the static $\bar{Q}Q$ potential, $V(\vec{x})$. $V(\vec{x})$ is related to the correlator of Polyakov lines $D(\vec{x})$

$$D(\vec{x}) = \langle L(\vec{x})L^\dagger(0) \rangle \quad (30)$$

by the relation

$$V(\vec{x}) = -\frac{1}{aN_t} \ln(D(\vec{x})) \quad (31)$$

where aN_t is the extension of the lattice in the time direction. By definition the Polyakov line is the trace

of the parallel transport across the lattice in the time direction.

$$L(\vec{x}) = P \exp[i \int_0^{1/T} d\tau A^0(\vec{x}, \tau)] \quad (32)$$

By cluster property

$$D(\vec{x}) \underset{x \rightarrow \infty}{\simeq} \exp(-\sigma x/T) + |\langle L \rangle|^2 \quad (33)$$

If $\langle L \rangle = 0$ it follows

$$V(\vec{x}) \sim \sigma x \quad \text{as } x \rightarrow \infty \quad (\text{confinement}) \quad (34)$$

If $\langle L \rangle \neq 0$

$$V(\vec{x}) \sim \text{const.} \quad \text{as } x \rightarrow \infty \quad (\text{deconfinement}) \quad (35)$$

A transition is observed on the lattice at $T_c \approx 270 \text{ MeV}$ from a region $T < T_c$ where $\langle L \rangle = 0$ to a region $T > T_c$ where $\langle L \rangle \neq 0$. A finite size scaling analysis allows to determine the order of the transition, which for SU(3) is first[31][30]. $\langle L \rangle$ can be then assumed as an order parameter for confinement, and the corresponding symmetry is Z_3

In principle one should show that $\langle L \rangle = 0$ implies absence of colored particles in all asymptotic states to prove confinement, which has not been done, but the criterion is reasonable anyhow.

In the presence of dynamical quarks (full QCD) Z_3 is not a symmetry anymore and $\langle L \rangle$ is not an order parameter. String breaking is expected to take place: when pulling the static $\bar{Q}Q$ pair apart from each other energy is converted into light $\bar{q}q$ pairs, and the potential is not linear any more, even if there is confinement.

At $m_q = 0$ another symmetry is present, chiral symmetry, which is spontaneously broken at $T = 0$, and, as lattice data show, is restored at some T_c . Chiral symmetry, however, is explicitly broken by quark masses, and cannot be the symmetry responsible for confinement.

Let us consider the case $N_f = 2$, $m_u = m_d = m$, which is semirealistic, but, as we shall see, very instructive.

A phase diagram is usually drawn for this system as in fig(4). The line which joins the first order phase transition at $m = \infty$ (quenched) to the chiral transition at $m = 0$ corresponds to the values $T_c(m)$ at which the susceptibilities C_V (specific heat) and $\chi_{\bar{\psi}\psi} = \int d^3x \langle \bar{\psi}(\vec{x})\psi(\vec{x})\bar{\psi}(0)\psi(0) \rangle$ are maximum[32][33].

Conventionally the region below this line is called confined, the region above it deconfined. No criterion exists which justifies this attribution.

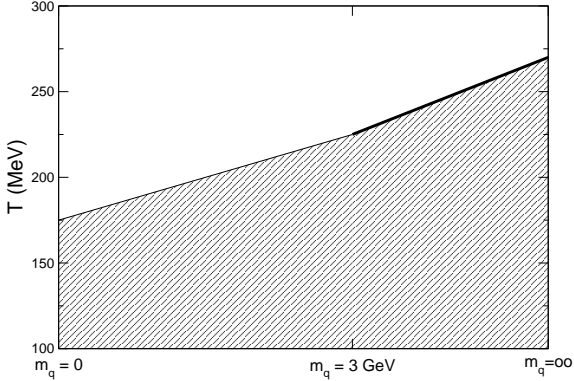


Figure 4. Schematic phase diagram of $N_f = 2$ QCD. The line corresponds to maxima of the susceptibilities.

An analysis can be made of the chiral phase transition at $m = 0$, assuming that the pions and sigmas are the relevant degrees of freedom [34]. Chiral symmetry fixes then the form of the free energy and renormalization group arguments allow to predict the order of the transition and the corresponding universality class.

For $N_f = 3$ the transition is first order. For $N_f = 2$ the order depends on the relative weight of the chiral $O(4)$ part of the action and the term describing the $U_A(1)$ anomaly. If the anomaly disappears below T_c the transition is first order, and such is the transition along the line at $m \neq 0$. If instead the anomaly persists up to T_c the transition is second order the universality class is $O(4)$ and the line at $m \neq 0$ is a crossover. A reliable numerical analysis of this issue is not yet available, but for some reason the second option is more popular.

The order and the universality class can be investigated by studying the behaviour of the critical specific heat as the volume goes to infinity by a finite size scaling analysis. A good order parameter should have a behaviour in that limit consistent with that of C_V . The order of the transition and the universality class should be identified by the symmetry responsible for confinement.

A valid candidate symmetry is dual superconductivity of the vacuum [35][36][37]. The basic idea is that the chromoelectric field acting between colored particles is channeled into dual Abrikosov flux tubes, whose energy is proportional to the length. Here dual means interchange of electric with magnetic with respect to ordinary superconductors.

This mechanism has been assumed as a working hypothesis and analyzed down to observable consequences in a series of papers [38][39][40][41][42][43][44][45].

A disorder parameter $\langle \mu \rangle$ has been defined which detects dual superconductivity: it is the vev of an op-

erator μ carrying magnetic charge which has zero vev in a phase in which vacuum has a definite magnetic charge, and can have a non zero vev if monopoles condense i.e. the vacuum is a superposition of states with different magnetic charge.

That operator is color gauge invariant, magnetically charged but magnetic $U(1)$ gauge invariant [11].

In principle $\langle \mu \rangle \neq 0$ or $\langle \mu \rangle = 0$ i.e. superconductivity or not depends on the procedure used to identify magnetic charges (Abelian Projection). However it can be shown that monopole condensation (or non condensation) is an abelian projection independent statement [44][45].

For quenched theory the result of numerical simulations is that this disorder parameter is consistent with $\langle L \rangle$ [41][42]: it is non zero for $T < T_c$ and strictly zero for $T > T_c$. By finite size scaling analysis of the corresponding susceptibility $\rho = \frac{d}{d\beta} \log(\langle \mu \rangle)$ the same critical indices are obtained as with $\langle L \rangle$.

The operator μ is perfectly well defined also in the presence of dynamical quarks, and can be used to investigate the phase diagram of fig(4). The result is that the region below the line is really confined, the one above it is deconfined [46].

ρ as a function of T at fixed m has a peak at the same value as the other susceptibilities, i.e. on the line of fig(4). $\langle \mu \rangle$ thus provides a good criterion for confinement.

The finite size scaling analysis works as follows. A priori $\langle \mu \rangle$ depends on the coupling constant β i.e. on the lattice spacing a , on the mass m and on the lattice size N_s . The dependence on β can be traded with the dependence on the correlation length ξ so that, for dimensional reasons

$$\langle \mu \rangle = \Phi\left(\frac{a}{\xi}, \frac{N_s}{\xi}, mN_s^\gamma\right) \quad (36)$$

As T approaches T_c from below, or $\tau \equiv (1 - \frac{T}{T_c}) \rightarrow 0$, ξ diverges as

$$\xi \propto \tau^{-\nu} \quad (37)$$

ν and γ are critical indices.

If the transition is second order or weak first order ξ goes large as T_c is approached and $\frac{a}{\xi}$ can be put equal to zero. Because of eq(37) $\frac{N_s}{\xi}$ can be traded with $\tau N_s^{1/\nu}$. The scaling law follows

$$\rho/N_s^{1/\nu} = f(\tau N_s^{1/\nu}, mN_s^\gamma) \quad (38)$$

In the quenched case the second dependence does not exist. The scaling can be tested on the lattice data and

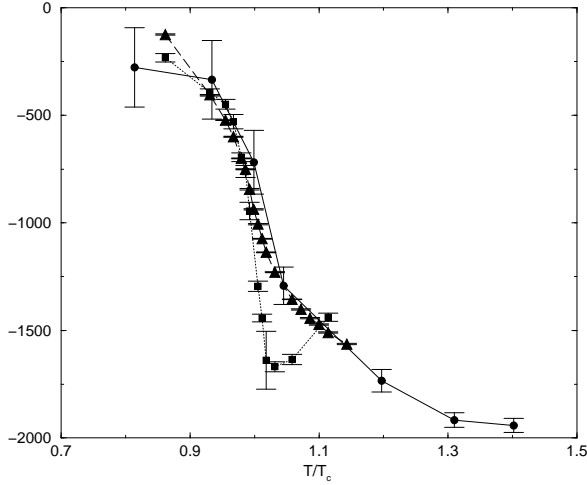


Figure 5. The behaviour of the topological susceptibility (circles), of the magnetic monopole susceptibility ρ (squares), and of the chiral parameter $\langle \bar{\psi}\psi \rangle$ (triangles) versus T/T_c at the deconfining transition.

the critical index ν can be extracted[41][42]. In particular for the peak one finds

$$\rho_{peak} \propto N_s^{1/\nu} \quad (39)$$

For the case of full QCD the problem has two scales and the analysis is more complicated. Preliminary results[47] exclude the critical indices of $O(4)$ and seem consistent with a first order phase transition. If this will be confirmed by the additional data in preparation either the anomaly disappears below T_c , or the relevant degrees of freedom which dominate are not the pion and the sigma[34].

The first possibility can be checked on the lattice. The dependence of the topological susceptibility at the deconfining transition has been studied[24]. Fig(5) shows the existing data. A more precise analysis is needed to settle the problem.

In conclusion there is strong evidence that dual superconductivity is the mechanism of confinement. A candidate disorder parameter exists, which is well defined and seems to work. The corresponding symmetry is dual superconductivity, which is however not clearly understood formally, being a common feature of all the abelian projections: consequently the effective action is not known.

A definite settlement of the order of the transition for the case $N_f = 2$ will be a crucial test.

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